4 Quadratic Equations

Lesson at a Glance

1. The standard form of a quadratic equation is

$$ax^2 + bx + c = 0, a \neq 0.$$

2. A quadratic equation has at most two roots.

3. A real number α is said to be a root of the quadratic equation $ax^2 + bx + c = 0$, if $a\alpha^2 + b\alpha + c = 0$.

4. If α and β are two roots of a quadratic equation

$$ax^2 + bx + c = 0, \text{ then}$$

Sum of the roots $= \alpha + \beta = -\frac{b}{a}$ and product of the roots

$$= \alpha\beta = \frac{c}{a}.$$

5. Discriminant of a quadratic equation $ax^2 + bx + c = 0$ is given by

$$D = b^2 - 4ac.$$

6. The equation $ax^2 + bx + c = 0$ has
   (*i*) two distinct real roots, if $b^2 - 4ac > 0$
   (*ii*) two equal real roots, if $b^2 - 4ac = 0$
   (*iii*) no real roots, if $b^2 - 4ac < 0$.

7. The roots of a quadratic equation $ax^2 + bx + c = 0$ are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

8. A quadratic equation can also be solved by the method of completing the square or by factorization.

9. In the method of factorization, we generally split the linear (middle) term into two parts.

10. If α and β are the two roots of a quadratic equation, then the quadratic equation will be

$$k[x^2 - (\alpha + \beta)x + \alpha\beta] = 0$$

where k is a non-zero real number.

11. If the polynomial corresponding to a quadratic equation is a perfect square, then its roots are equal.

12. If each root of a quadratic equation

$ax^2 + bx + c = 0$ is zero, then $b = c = 0$.

TEXTBOOK QUESTIONS SOLVED

Exercise 4.1 *(Page – 73-74)*

1. *Check whether the following are quadratic equations:*

(i) $(x + 1)^2 = 2(x - 3)$

(ii) $x^2 - 2x = (-2)(3 - x)$

(iii) $(x - 2)(x + 1) = (x - 1)(x + 3)$

(iv) $(x - 3)(2x + 1) = x(x + 5)$

(v) $(2x - 1)(x - 3) = (x + 5)(x - 1)$

(vi) $x^2 + 3x + 1 = (x - 2)^2$

(vii) $(x + 2)^3 = 2x(x^2 - 1)$

(viii) $x^3 - 4x^2 - x + 1 = (x - 2)^3$

Sol. (i) $(x + 1)^2 = 2(x - 3) \Rightarrow x^2 + 2x + 1 = 2x - 6$

$\Rightarrow x^2 + 7 = 0.$

The equation is of the form $ax^2 + bx + c = 0, a \neq 0$.

Hence, it is a quadratic equation.

(ii) $x^2 - 2x = (-2)(3 - x) \Rightarrow x^2 - 2x = -6 + 2x$

$\Rightarrow x^2 - 4x + 6 = 0.$

The equation is of the form $ax^2 + bx + c = 0, a \neq 0$.

Hence, it is a quadratic equation.

(iii) $(x - 2)(x + 1) = (x - 1)(x + 3)$

$\Rightarrow x^2 - x - 2 = x^2 + 2x - 3 \quad \Rightarrow \quad 3x - 1 = 0.$

The equation is not of the form

$ax^2 + bx + c = 0, a \neq 0$.

Hence, it is not a quadratic equation.

(iv) $(x - 3)(2x + 1) = x(x + 5)$

$\Rightarrow \quad 2x^2 + x - 6x - 3 = x^2 + 5x$

$\Rightarrow \quad x^2 - 10x - 3 = 0.$

The equation is of the form $ax^2 + bx + c = 0, a \neq 0$.

Hence, it is a quadratic equation.

(*v*) $(2x - 1)(x - 3) = (x + 5)(x - 1)$

$\Rightarrow \quad 2x^2 - 7x + 3 = x^2 + 4x - 5$

$\Rightarrow \quad x^2 - 11x + 8 = 0.$

The equation is of the form $ax^2 + bx + c = 0, a \neq 0$.

Hence, it is a quadratic equation.

(*vi*) $x^2 + 3x + 1 = (x - 2)^2$

$\Rightarrow \quad x^2 + 3x + 1 = x^2 - 4x + 4$

$\Rightarrow \quad 7x - 3 = 0.$

The equation is not of the form

$ax^2 + bx + c = 0, a \neq 0.$

Hence, it is not a quadratic equation.

(*vii*) $(x + 2)^3 = 2x(x^2 - 1)$

$\Rightarrow \quad x^3 + 6x^2 + 12x + 8 = 2x^3 - 2x$

$\Rightarrow \quad x^3 - 6x^2 - 14x - 8 = 0.$

The equation is not of the form

$ax^2 + bx + c = 0, a \neq 0.$

Hence, it is not a quadratic equation.

(*viii*) $x^3 - 4x^2 - x + 1 = (x - 2)^3$

$\Rightarrow \quad x^3 - 4x^2 - x + 1 = x^3 - 6x^2 + 12x - 8$

$\Rightarrow \quad 2x^2 - 13x + 9 = 0$

The equation is of the form $ax^2 + bx + c = 0, a \neq 0$.

Hence, it is a quadratic equation.

2. *Represent the following situations in the form of quadratic equations:*

   (*i*) *The area of a rectangular plot is 528 m^2. The length of the plot (in metres) is one more than twice its breadth. We need to find the length and breadth of the plot.*

   (*ii*) *The product of two consecutive positive integers is 306. We need to find the integers.*

(*iii*) *Rohan's mother is 26 years older than him. The product of their ages (in years) 3 years from now will be 360. We would like to find Rohan's present age.*

(*iv*) *A train travels a distance of 480 km at a uniform speed. If the speed had been 8 km/h less, then it would have taken 3 hours more to cover the same distance. We need to find the speed of the train.*

Sol. (*i*) Let breadth of plot = x m. Then length = $(2x + 1)$ m

According to given condition,

$$(2x + 1)x = 528$$

$$\Rightarrow \quad 2x^2 + x - 528 = 0$$

is the required quadratic equation.

(*ii*) Let two consecutive positive integers be x and $x + 1$.

According to the given condition,

$$\therefore \quad x(x + 1) = 306$$

$$\Rightarrow \quad x^2 + x - 306 = 0$$

is the required quadratic equation.

(*iii*) Let Rohan's present age = x years

Then his mother's present age = $(x + 26)$ years

3 years from now,

Rohan's age = $(x + 3)$ years

and his mother's age = $(x + 26 + 3)$ years = $(x + 29)$ years

According to the given condition,

$$(x + 3)(x + 29) = 360 \quad \Rightarrow x^2 + 32x + 87 = 360$$

$$\Rightarrow \quad x^2 + 32x - 273 = 0$$

is the required quadratic equation.

(*iv*) Let speed of the train = x km/hr

Distance = 480 km

$$\therefore \quad \text{Time taken} = \frac{480}{x} \text{ hours} \quad ...(i)$$

New speed = $(x - 8)$ km/hr

New time taken $= \dfrac{480}{x-8}$ hours ...(*ii*)

As new time taken is greater than the usual time by 3 hours, therefore, from (*i*) and (*ii*), we have

$$\frac{480}{x-8} - \frac{480}{x} = 3 \Rightarrow 480x - 480x + 3840 = 3(x)(x-8)$$

$\Rightarrow$ $x^2 - 8x - 1280 = 0$ is the required quadratic equation.

Exercise 4.2 *(Page – 76)*

1. *Find the roots of the following quadratic equations by factorisation:*

   (*i*) $x^2 - 3x - 10 = 0$ (*ii*) $2x^2 + x - 6 = 0$

   (*iii*) $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$ (*iv*) $2x^2 - x + \dfrac{1}{8} = 0$

   (*v*) $100x^2 - 20x + 1 = 0$

Sol. (*i*) Consider $x^2 - 3x - 10 = 0$

$\Rightarrow x^2 - 5x + 2x - 10 = 0$

$\Rightarrow x(x-5) + 2(x-5) = 0$

$\Rightarrow (x-5)(x+2) = 0$

$\Rightarrow x - 5 = 0$ or $x + 2 = 0$

$\Rightarrow x = 5, -2.$

(*ii*) Consider $2x^2 + x - 6 = 0$

$\Rightarrow 2x^2 + 4x - 3x - 6 = 0$

$\Rightarrow 2x(x+2) - 3(x+2) = 0$

$\Rightarrow (2x-3)(x+2) = 0$

$\Rightarrow 2x - 3 = 0$ or $x + 2 = 0$

$\Rightarrow x = \dfrac{3}{2}, -2.$

(*iii*) Consider $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$

$\Rightarrow \sqrt{2}x^2 + 5x + 2x + 5\sqrt{2} = 0$

$\Rightarrow x(\sqrt{2}x + 5) + \sqrt{2}(\sqrt{2}x + 5) = 0$

$$\Rightarrow \quad (x + \sqrt{2})(\sqrt{2}x + 5) = 0$$

$$\Rightarrow \quad x + \sqrt{2} = 0 \text{ or } \sqrt{2}x + 5 = 0$$

$$\Rightarrow \quad x = -\sqrt{2}, -\frac{5}{\sqrt{2}}.$$

(*iv*) Consider $2x^2 - x + \frac{1}{8} = 0 \Rightarrow 16x^2 - 8x + 1 = 0$

$$\Rightarrow \quad 16x^2 - 4x - 4x + 1 = 0$$

$$\Rightarrow \quad 4x(4x - 1) - 1(4x - 1) = 0$$

$$\Rightarrow \quad (4x - 1)(4x - 1) = 0$$

$$\Rightarrow \quad 4x - 1 = 0 \text{ or } 4x - 1 = 0$$

$$\Rightarrow \quad x = \frac{1}{4}, \frac{1}{4}.$$

(*v*) Consider $100x^2 - 20x + 1 = 0$

$$\Rightarrow \quad 100x^2 - 10x - 10x + 1 = 0$$

$$\Rightarrow \quad 10x(10x - 1) - 1(10x - 1) = 0$$

$$\Rightarrow \quad (10x - 1)^2 = 0$$

$$\Rightarrow \quad 10x - 1 = 0 \text{ or } 10x - 1 = 0$$

$$\Rightarrow \quad x = \frac{1}{10}, \frac{1}{10}.$$

2. *Solve the problems given below:*

(*i*) *John and Jivanti together have 45 marbles. Both of them lost 5 marbles each, and the product of the number of marbles they now have is 124. Find the number of marbles each of them had to start with.*

(*ii*) *A cottage industry produces a certain number of toys in a day. The cost of production of each toy (in rupees) was found to be 55 minus the number of toys produced in a day. On a particular day, the total cost of production was ₹ 750. Find the number of toys produced on that day.*

Sol. (*i*) Let one of them had x marbles, then the other one had $(45 - x)$ marbles.

$$\therefore \quad (x - 5)(45 - x - 5) = 124$$

$$\Rightarrow \quad (x - 5)(40 - x) = 124$$

$$x^2 - 45x + 324 = 0$$

$$\Rightarrow \quad x^2 - 9x - 36x + 324 = 0$$

$$\Rightarrow \quad x(x - 9) - 36(x - 9) = 0 \Rightarrow (x - 9)(x - 36) = 0$$

$$\Rightarrow \quad x - 9 = 0, x - 36 = 0 \Rightarrow x = 9, 36.$$

Hence if John had 9 marbles then Jivanti had 36 marbles and if John had 36 marbles then Jivanti had 9 marbles.

(*ii*) Let the number of toys produced on that day be x. Then

cost of 1 toy = ₹ $\dfrac{750}{x}$

According to the given condition,

$$\frac{750}{x} = 55 - x \quad \Rightarrow \quad 750 = 55x - x^2$$

$$x^2 - 55x + 750 = 0 \Rightarrow x^2 - 25x - 30x + 750 = 0$$

$$\Rightarrow \quad x(x - 25) - 30(x - 25) = 0$$

$$\Rightarrow \quad (x - 30)(x - 25) = 0$$

$$\Rightarrow \quad x - 30 = 0, x - 25 = 0 \quad \Rightarrow \quad x = 30, 25.$$

Hence number of toys produced is 30 or 25.

3. *Find two numbers whose sum in 27 and product is 182.*

Sol. Let numbers be x and y.

According to given condition,

$$x + y = 27 \quad ...(i) \quad \text{and} \quad xy = 182 \quad ...(ii)$$

From (*i*) and (*ii*), we get

$$x(27 - x) = 182 \Rightarrow x^2 - 27x + 182 = 0$$

$$\Rightarrow \quad x^2 - 14x - 13x + 182 = 0$$

$$\Rightarrow \quad x(x - 14) - 13(x - 14) = 0 \Rightarrow (x - 13)(x - 14) = 0$$

$$\Rightarrow \quad x - 13 = 0 \text{ or } x - 14 = 0 \Rightarrow x = 13 \text{ or } 14$$

when $x = 13, y = 14$ and when $x = 14, y = 13$

Therefore, the numbers are 13 and 14.

4. *Find two consecutive positive integers, sum of whose squares is 365.*

Sol. Let numbers be x and $x + 1$.

According to given condition,

$$x^2 + (x + 1)^2 = 365 \Rightarrow 2x^2 + 2x - 364 = 0$$

$$\Rightarrow x^2 + x - 182 = 0 \Rightarrow x^2 + 14x - 13x - 182 = 0$$

$$\Rightarrow x(x + 14) - 13(x + 14) = 0 \Rightarrow (x + 14)(x - 13) = 0$$

$$\Rightarrow x + 14 = 0 \quad \text{or} \quad x - 13 = 0$$

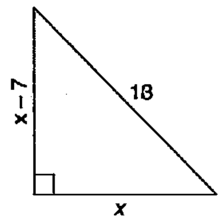
$$\Rightarrow x = -14 \text{ (rejected)}, \quad x = 13.$$

Hence, positive integers are 13, 14.

5. *The altitude of a right triangle is 7 cm less than its base. If the hypotenuse is 13 cm, find the other two sides.*

Sol. Let base = x cm. Then altitude = $(x - 7)$ cm

Also hypotenuse = 13 cm



$$\therefore \quad x^2 + (x - 7)^2 = (13)^2$$

[Using Pythagoras theorem]

$$\Rightarrow x^2 + x^2 - 14x + 49 = 169$$

$$\Rightarrow 2x^2 - 14x - 120 = 0$$

$$\Rightarrow x^2 - 7x - 60 = 0$$

$$\Rightarrow (x - 12)(x + 5) = 0 \Rightarrow x - 12 = 0 \quad \text{or} \quad x + 5 = 0$$

$$\Rightarrow x = 12 \quad \text{or} \quad x = -5 \text{ (rejected)}$$

$\therefore$ Base = 12 cm, altitude = 5 cm.

6. *A cottage industry produces a certain number of pottery articles in a day. It was observed on a particular day that the cost of production of each article (in rupees) was 3 more than twice the number of articles produced on that day. If the total cost of production on that day was ₹ 90, find the number of articles produced and the cost of each article.*

Sol. Let number of articles produced = x.

Then cost of production for each article = $(2x + 3)$

According to the given condition,

$$(2x + 3)x = 90 \Rightarrow 2x^2 + 3x - 90 = 0$$

$$\Rightarrow 2x^2 + 15x - 12x - 90 = 0$$

$$\Rightarrow x(2x + 15) - 6(2x + 15) = 0$$

$$\Rightarrow (2x + 15)(x - 6) = 0$$

$$\Rightarrow 2x + 15 = 0 \quad \text{or} \quad x - 6 = 0$$

$$\Rightarrow x = -\frac{15}{2} \text{ (rejected) or } x = 6.$$

$\therefore$ Number of articles produced = 6 and cost of each article = ₹ 15.

Exercise 4.3 *(Page – 87-88)*

1. *Find the roots of the following quadratic equations, if they exist, by the method of completing the square:*

   (i) $2x^2 - 7x + 3 = 0$ (ii) $2x^2 + x - 4 = 0$

   (iii) $4x^2 + 4\sqrt{3}x + 3 = 0$ (iv) $2x^2 + x + 4 = 0$

Sol. (i) Consider equation $2x^2 - 7x + 3 = 0$

$\Rightarrow 4x^2 - 14x + 6 = 0$

[On multiplying throughout by 2]

$$\Rightarrow (2x)^2 - 2 \cdot \frac{7}{2} \cdot (2x) + \left(\frac{7}{2}\right)^2 - \left(\frac{7}{2}\right)^2 + 6 = 0$$

$$\Rightarrow \left(2x - \frac{7}{2}\right)^2 - \frac{49}{4} + 6 = 0 \Rightarrow \left(2x - \frac{7}{2}\right)^2 = \frac{25}{4}$$

$$\Rightarrow 2x - \frac{7}{2} = \pm \frac{5}{2} \Rightarrow 2x = \frac{7}{2} \pm \frac{5}{2}$$

$$\Rightarrow 2x = \frac{7}{2} + \frac{5}{2} \text{ or } 2x = \frac{7}{2} - \frac{5}{2}$$

$$\Rightarrow x = 3 \text{ or } x = \frac{1}{2}$$

Hence 3 and $\frac{1}{2}$ are the required roots.

(ii) Consider equation $2x^2 + x - 4 = 0$

$\Rightarrow 4x^2 + 2x - 8 = 0$

[On multiplying throughout by 2]

$$\Rightarrow (2x)^2 + 2 \,.\, (2x) \,.\, \frac{1}{2} + \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2 - 8 = 0$$

$$\Rightarrow \left(2x + \frac{1}{2}\right)^2 = \frac{33}{4} \Rightarrow 2x + \frac{1}{2} = \pm \frac{\sqrt{33}}{2}$$

$$\Rightarrow 2x = -\frac{1}{2} \pm \frac{\sqrt{33}}{2}$$

$$\Rightarrow \quad x = \frac{-1+\sqrt{33}}{4}, \frac{-1-\sqrt{33}}{4} \text{ are the required roots.}$$

(*iii*) Consider equation $4x^2 + 4\sqrt{3}\,x + 3 = 0$

$$\Rightarrow \quad (2x)^2 + 2\cdot(\sqrt{3})(2x) + (\sqrt{3})^2 - (\sqrt{3})^2 + 3 = 0$$

$$\Rightarrow \quad (2x + \sqrt{3})^2 = 0 \quad \Rightarrow \quad (2x + \sqrt{3})^2 = 0$$

$$\Rightarrow \quad 2x + \sqrt{3} = 0 \text{ or } 2x + \sqrt{3} = 0$$

$$\Rightarrow \quad x = -\frac{\sqrt{3}}{2} \text{ and } x = -\frac{\sqrt{3}}{2} \text{ are the required roots.}$$

(*iv*) Consider equation $2x^2 + x + 4 = 0$

$$\Rightarrow \quad 4x^2 + 2x + 8 = 0$$

[On multiplying throughout by 2]

$$\Rightarrow \quad (2x)^2 + 2\,.\,(2x)\,.\,\frac{1}{2} + \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2 + 8 = 0$$

$$\Rightarrow \quad \left(2x+\frac{1}{2}\right)^2 = -\frac{31}{4}$$

This equation is not valid as a square can never be negative in real number. Hence, no root exist.

2. *Find the roots of the quadratic equations given in Q.1 above by applying the quadratic formula.*

Sol. (*i*) Consider equation $2x^2 - 7x + 3 = 0$

Here, $a = 2,\ b = -7,\ c = 3$

$$\therefore \quad D = b^2 - 4ac = (-7)^2 - 4 \times 2 \times 3$$

$$= 49 - 24 = 25 > 0$$

$$\therefore \quad \text{Roots are} \quad \alpha = \frac{-b+\sqrt{D}}{2a} = \frac{-(-7)+\sqrt{25}}{2\times 2}$$

$$= \frac{7+5}{4} = 3$$

$$\text{and} \quad \beta = \frac{-b-\sqrt{D}}{2a} = \frac{-(-7)-\sqrt{25}}{2\times 2} = \frac{7-5}{4} = \frac{1}{2}$$

$\therefore$ Roots are 3 and $\frac{1}{2}$.

(*ii*) Consider equation $2x^2 + x - 4 = 0$

Here, $a = 2, b = 1, c = -4$

$$D = b^2 - 4ac = (1)^2 - 4 \times 2 \times (-4)$$
$$= 1 + 32 = 33 > 0$$

$\therefore$ Roots are: $\dfrac{-1+\sqrt{33}}{4}$ and $\dfrac{-1-\sqrt{33}}{4}$.

(*iii*) Consider equation $4x^2 + 4\sqrt{3}x + 3 = 0$

Here $a = 4, b = 4\sqrt{3}, c = 3$

$\therefore$ $$D = b^2 - 4ac = (4\sqrt{3})^2 - 4 \times 4 \times 3$$
$$= 48 - 48 = 0$$

$\therefore$ Roots are

$$\alpha = \frac{-b+\sqrt{D}}{2a} = \frac{-4\sqrt{3} + \sqrt{0}}{2 \times 4} = \frac{-4\sqrt{3}}{8} = -\frac{\sqrt{3}}{2}$$

and $$\beta = \frac{-b-\sqrt{D}}{2a} = \frac{-4\sqrt{3} - \sqrt{0}}{2 \times 4} = \frac{-4\sqrt{3}}{8} = -\frac{\sqrt{3}}{2}$$

$\therefore$ Required roots are $-\dfrac{\sqrt{3}}{2}$ and $-\dfrac{\sqrt{3}}{2}$.

(*iv*) Consider equation $2x^2 + x + 4 = 0$

Here, $a = 2, b = 1, c = 4$

$\therefore$ $$D = b^2 - 4ac = (1)^2 - 4 \times 2 \times 4$$
$$= 1 - 32 = -31 < 0$$

As $D < 0$, no real roots.

3. *Find the roots of the following equations:*

(*i*) $x - \dfrac{1}{x} = 3, x \neq 0$

(*ii*) $\dfrac{1}{x+4} - \dfrac{1}{x-7} = \dfrac{11}{30}, x \neq -4, 7.$

Sol. (*i*) Consider equation $x - \dfrac{1}{x} = 3; x \neq 0$

$\Rightarrow x^2 - 3x - 1 = 0$

Here $a = 1, b = -3, c = -1$

$\therefore$ $$D = b^2 - 4ac = (-3)^2 - 4 \times 1 \times (-1)$$
$$= 9 + 4 = 13 > 0$$

$$\therefore \quad \text{Roots are } \alpha = \frac{-b+\sqrt{D}}{2a} = \frac{-(-3)+\sqrt{13}}{2 \times 1}$$

$$= \frac{3+\sqrt{13}}{2}$$

$$\text{and} \quad \beta = \frac{-b-\sqrt{D}}{2a} = \frac{-(-3)-\sqrt{13}}{2 \times 1} = \frac{3-\sqrt{13}}{2}$$

$$\therefore \quad \text{Roots are } \frac{3+\sqrt{13}}{2} \text{ and } \frac{3-\sqrt{13}}{2}.$$

(*ii*) Consider equation $\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}$; $x \neq -4, 7$

$$\Rightarrow \frac{x-7-x-4}{(x+4)(x-7)} = \frac{11}{30} \quad \Rightarrow \quad \frac{-11}{x^2-3x-28} = \frac{11}{30}$$

$$\Rightarrow \quad x^2 - 3x - 28 = -30 \Rightarrow x^2 - 3x + 2 = 0$$

$$\Rightarrow \quad (x-1)(x-2) = 0$$

$$\Rightarrow \quad x - 1 = 0 \text{ or } x - 2 = 0 \Rightarrow x = 1 \text{ or } x = 2$$

$\Rightarrow$ 1 and 2 are the required roots.

4. *The sum of the reciprocals of Rehman's age, (in years) 3 years ago and 5 years from now is $\frac{1}{3}$. Find his present age.*

Sol. Let Rehman's present age = x years

Age 3 years ago = $(x - 3)$ years

Age 5 years from now = $(x + 5)$ years

According to given condition,

$$\frac{1}{x-3} + \frac{1}{x+5} = \frac{1}{3}$$

$$\Rightarrow \frac{x+5+x-3}{(x-3)(x+5)} = \frac{1}{3} \quad \Rightarrow \quad \frac{2x+2}{x^2+2x-15} = \frac{1}{3}$$

$$\Rightarrow \quad x^2 + 2x - 15 = 6x + 6 \quad \Rightarrow \quad x^2 - 4x - 21 = 0$$

$$\Rightarrow \quad (x-7)(x+3) = 0 \Rightarrow x - 7 = 0 \text{ or } x + 3 = 0$$

$$\Rightarrow \quad x = 7 \text{ or } x = -3 \text{ (rejected)}$$

$\therefore$ Rehman's present age = 7 years.

5. *In a class test, the sum of Shefali's marks in Mathematics and English is 30. Had she got 2 marks more in Mathematics and 3 marks less in English, the product of their marks would have been 210. Find her marks in the two subjects.*

Sol. Let marks in Mathematics = x and marks in English = y

According to given conditions,

$$x + y = 30 \quad ...(i)$$

and $$(x + 2)(y - 3) = 210 \quad ...(ii)$$

From (*i*) and (*ii*), we have

$$(x + 2)(30 - x - 3) = 210 \Rightarrow (x + 2)(27 - x) = 210$$

$$\Rightarrow \quad 27x - x^2 + 54 - 2x = 210 \Rightarrow x^2 - 25x + 156 = 0$$

$$\Rightarrow \quad (x - 13)(x - 12) = 0$$

$$\Rightarrow \quad x - 13 = 0 \quad \text{or} \quad x - 12 = 0$$

$$\Rightarrow \quad x = 13 \quad \text{or} \quad x = 12$$

When $x = 13, y = 17$ and when $x = 12, y = 18$

Hence, marks in Mathematics are 13 and in English are 17 or marks in Mathematics are 12 and in English are 18.

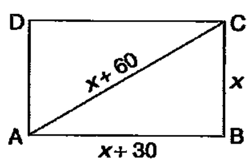
6. *The diagonal of a rectangular field is 60 metres more than the shorter side. If the longer side is 30 metres more than the shorter side, find the sides of the field.*

Sol. Let shorter side of rectangle = x m.

Then longer side = $(x + 30)$ m and diagonal = $(x + 60)$ m

Using Pythagoras theorem, we have

$$AC^2 = AB^2 + BC^2 \text{ (see figure)}$$



$$(x + 60)^2 = (x + 30)^2 + x^2$$

$$\Rightarrow \quad x^2 + 120x + 3600 = x^2 + 60x + 900 + x^2$$

$$\Rightarrow \quad x^2 - 60x - 2700 = 0 \Rightarrow (x - 90)(x + 30) = 0$$

$$\Rightarrow \quad x - 90 = 0 \quad \text{or} \quad x + 30 = 0$$

$\Rightarrow \quad x = 90 \text{ or } -30$ (rejected)

Therefore, shorter side = 90 m, longer side = 120 m.

7. *The difference of squares of two numbers is 180. The square of the smaller number is 8 times the larger number. Find the two numbers.*

Sol. Let smaller number = x and larger number = y, $y > x$

According to the given conditions,

$$y^2 - x^2 = 180 \quad \text{...}(i) \quad [\text{As } y > x]$$

and
$$x^2 = 8y \quad \text{...}(ii)$$

From (*i*) and (*ii*), we get

$$y^2 - 8y = 180 \Rightarrow y^2 - 8y - 180 = 0$$

$\Rightarrow \quad y^2 - 18y + 10y - 180 = 0$

$\Rightarrow \quad y(y - 18) + 10(y - 18) = 0 \quad \Rightarrow (y + 10)(y - 18) = 0$

$\Rightarrow \quad y + 10 = 0 \quad \Rightarrow \quad y = -10$ (rejected) [From (*ii*)]

or $\quad y - 18 = 0 \quad \Rightarrow \quad y = 18$

From (*ii*), we get $\quad x^2 = 8 \times 18 = 144 \quad \Rightarrow \quad x = \pm 12$

Hence, numbers are 18, 12 or 18, – 12.

8. *A train travels 360 km at a uniform speed. If the speed had been 5 km/h more, it would have taken 1 hour less for the same journey. Find the speed of the train.*

Sol. Let uniform speed of the train = x km / hr

According to the given condition,

$$\frac{360}{x} - \frac{360}{x+5} = 1$$

$\Rightarrow \quad 360x + 1800 - 360x = x^2 + 5x$

$\Rightarrow \quad x^2 + 5x - 1800 = 0$

$\Rightarrow \quad x^2 + 45x - 40x - 1800 = 0$

$\Rightarrow \quad x(x + 45) - 40(x + 45) = 0$

$\Rightarrow \quad (x - 40)(x + 45) = 0$

$\Rightarrow \quad x - 40 = 0 \text{ or } x + 45 = 0$

$\Rightarrow \quad x = 40 \text{ or } x = -45$ (rejected)

$\therefore$ Speed of the train = 40 km/hr.

9. *Two water taps together can fill a tank in $9\frac{3}{8}$ hours. The tap of larger diameter takes 10 hours less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank.*

Sol. Let time taken by smaller tap = x hours

Then time taken by larger tap = $(x - 10)$ hours

In 1 hour, smaller tap fills $\frac{1}{x}$ of the tank

In 1 hour, larger tap fills $\frac{1}{x-10}$ of the tank

Therefore, in 1 hour both can fill $\left(\frac{1}{x}+\frac{1}{x-10}\right)$ of the tank

According to the given condition,

$$\frac{75}{8}\left(\frac{1}{x}+\frac{1}{x-10}\right) = 1 \quad \text{(full tank)}$$

$$\Rightarrow \quad \frac{1}{x} + \frac{1}{x-10} = \frac{8}{75}$$

$$\Rightarrow \quad \frac{x - 10 + x}{x(x-10)} = \frac{8}{75} \Rightarrow \frac{2x - 10}{x^2 - 10x} = \frac{8}{75}$$

$$\Rightarrow \quad 8x^2 - 80x = 150x - 750$$

$$\Rightarrow \quad 8x^2 - 230x + 750 = 0$$

$$\Rightarrow \quad 4x^2 - 115x + 375 = 0$$

$$\Rightarrow \quad 4x^2 - 100x - 15x + 375 = 0$$

$$\Rightarrow \quad 4x(x - 25) - 15(x - 25) = 0$$

$$\Rightarrow \quad (4x - 15)(x - 25) = 0$$

$$\Rightarrow \quad 4x - 15 = 0 \text{ or } x - 25 = 0$$

$$\Rightarrow \quad x = \frac{15}{4} \text{ (rejected, as value is less than 10);} \quad x = 25$$

Therefore, smaller tap can fill in 25 hours and larger tap can fill in 15 hours.

10. *An express train takes 1 hour less than a passenger train to travel 132 km between Mysore and Bangalore (without taking into consideration the time they stop at intermediate stations). If the average speed of the express train is 11 km/h* *more than that of the passenger train, find the average speed of the two trains.*

Sol. Let speed of passenger train $= x$ km/hr

Then speed of express train $= (x + 11)$ km/hr

Total distance $= 132$ km

Time taken by passenger train $= \frac{132}{x}$ hr

and time taken by express train $= \frac{132}{x+11}$ hours

According to the given condition, $\frac{132}{x} - \frac{132}{x+11} = 1$

$$\Rightarrow \quad \frac{132x + 1452 - 132x}{x(x + 11)} = 1 \quad \Rightarrow \quad x^2 + 11x = 1452$$

$$\Rightarrow \quad x^2 + 11x - 1452 = 0 \quad \Rightarrow \quad (x + 44)(x - 33) = 0$$

$$\Rightarrow \quad x + 44 = 0 \quad \text{or} \quad x - 33 = 0$$

$$\Rightarrow \quad x = -44 \text{ (rejected)} \quad \text{or} \quad x = 33$$

$\therefore$ Speed of the passenger train $= 33$ km/hr

and the speed of the express train $= 44$ km/hr.

11. *Sum of the areas of two squares is 468 m^2. If the difference of their perimeters is 24 m, find the sides of the two squares.*

Sol. Let the sides of the two squares be x m and y m respectively.

According to the given condition,

$$x^2 + y^2 = 468 \quad \text{...(i)}$$

and $$4x - 4y = 24 \quad \text{or} \quad x - y = 6 \quad \text{...(ii)}$$

From (*i*) and (*ii*), we get

$$x^2 + (x - 6)^2 = 468$$

$$\Rightarrow \quad x^2 + x^2 - 12x + 36 = 468 \quad \Rightarrow \quad 2x^2 - 12x - 432 = 0$$

$$\Rightarrow \quad x^2 - 6x - 216 = 0 \quad \Rightarrow \quad (x - 18)(x + 12) = 0$$

$$\Rightarrow \quad x - 18 = 0 \quad \text{or} \quad x + 12 = 0$$

$$\Rightarrow \quad x = 18 \quad \text{or} \quad x = -12 \text{ (rejected)}$$

Substituting $x = 18$ in (*ii*), we get

$$y = 12$$

Thus, sides of the two squares are 18 m and 12 m.

Exercise 4.4 *(Page – 91)*

1. *Find the nature of the roots of the following quadratic equations. If the real roots exist, find them:*

(i) $2x^2 - 3x + 5 = 0$ (ii) $3x^2 - 4\sqrt{3}x + 4 = 0$

(iii) $2x^2 - 6x + 3 = 0$

Sol. (i) Consider equation $2x^2 - 3x + 5 = 0$

Here, $a = 2, b = -3, c = 5$

$\therefore$ $D = b^2 - 4ac = (-3)^2 - 4 \times 2 \times 5 = 9 - 40$

$= -31 < 0$

As $D < 0$, no real roots for the equation.

(ii) Consider equation $3x^2 - 4\sqrt{3}x + 4 = 0$

Here, $a = 3, b = -4\sqrt{3}, c = 4$

$\therefore$ $D = b^2 - 4ac = (-4\sqrt{3})^2 - 4 \times 3 \times 4$

$= 48 - 48 = 0$

As $D = 0$, therefore, roots are real and equal

Roots are $\alpha = \dfrac{-b + \sqrt{D}}{2a} = \dfrac{-(-4\sqrt{3}) + 0}{2 \times 3}$

$= \dfrac{4\sqrt{3}}{6} = \dfrac{2\sqrt{3}}{3}$

and $\beta = \dfrac{-b - \sqrt{D}}{2a} = \dfrac{-(-4\sqrt{3}) - 0}{2 \times 3}$

$= \dfrac{4\sqrt{3}}{6} = \dfrac{2\sqrt{3}}{3}$

Therefore, required roots are $\dfrac{2\sqrt{3}}{3}$ and $\dfrac{2\sqrt{3}}{3}$.

(iii) Consider equation $2x^2 - 6x + 3 = 0$

Here, $a = 2, b = -6, c = 3$

$\therefore$ $D = b^2 - 4ac = (-6)^2 - 4 \times 2 \times 3$

$= 36 - 24 = 12 > 0$

As $D > 0$, therefore, roots are real and unequal.

Roots are $x = \dfrac{-(-6) \pm \sqrt{12}}{4} = \dfrac{6 \pm 2\sqrt{3}}{4} = \dfrac{3 \pm \sqrt{3}}{2}$,

i.e., $\dfrac{3+\sqrt{3}}{2}$ and $\dfrac{3-\sqrt{3}}{2}$.

2. *Find the values of k for each of the following quadratic equations, so that they have two equal roots.*

(i) $2x^2 + kx + 3 = 0$ (ii) $kx(x - 2) + 6 = 0$

Sol. (i) Consider the equation $2x^2 + kx + 3 = 0$

Here, $a = 2, b = k, c = 3$

$\therefore \quad D = b^2 - 4ac = k^2 - 4 \times 2 \times 3 = k^2 - 24$

For real and equal roots, $D = 0 \Rightarrow k^2 - 24 = 0$

$\Rightarrow k^2 = 24 \Rightarrow k = \pm\sqrt{24}$

$\Rightarrow k = \pm 2\sqrt{6}$.

(ii) Consider the equation $kx(x - 2) + 6 = 0$

$\Rightarrow kx^2 - 2kx + 6 = 0$

Here, $a = k, b = -2k$ and $c = 6$

$\therefore \quad D = b^2 - 4ac = (-2k)^2 - 4 \times k \times 6 = 4k^2 - 24k$

For real and equal roots, $D = 0$

$\Rightarrow 4k^2 - 24k = 0 \Rightarrow 4k(k - 6) = 0$

$\Rightarrow k = 0$ or $k - 6 = 0$

$\Rightarrow k = 0$ or $k = 6$.

But $k = 0$ is not possible [From given equation]

$\therefore \quad k = 6$.

3. *Is it possible to design a rectangular mango grove whose length is twice its breadth, and the area is 800 m^2? If so, find its length and breadth.*

Sol. Let breadth = x m. Then length = $2x$ m

Also $x \times 2x = 800 \Rightarrow 2x^2 = 800 \Rightarrow x^2 = 400$

$\Rightarrow \quad x = 20$ (-20 is rejected)

It is possible to design a rectangular mango grove with length 40 m and breadth 20 m.

4. *Is the following situation possible? If so, determine their present ages. The sum of the ages of two friends is 20 years. Four years ago, the product of their ages in years was 48.*

Sol. Let present age of one friend = x years

and present age of other friend = y years

We have, $x + y = 20$ ...(i)

Four years ago,

$$\text{Friend's age} = (x - 4) \text{ years}$$

$$\text{Other friend's age} = (y - 4) \text{ years}$$

We have $(x - 4)(y - 4) = 48$

$\Rightarrow \quad xy - 4x - 4y + 16 = 48$

$\Rightarrow \quad xy - 4(x + y) - 32 = 0 \quad \Rightarrow \quad xy - 4(20) - 32 = 0$

$\Rightarrow \quad xy = 112$ ...(*ii*)

From (*i*) and (*ii*), we have

$x(20 - x) = 112 \quad \Rightarrow \quad x^2 - 20x + 112 = 0$

$\therefore \quad D = b^2 - 4ac = (-20)^2 - 4 \times 1 \times 112$

$= 400 - 448 = -48 < 0$

As $D < 0$, therefore, the equation has no real roots.

So, situation is not possible.

5. *Is it possible to design a rectangular park of perimeter 80 m and area 400 m²? If so, find its length and breadth.*

Sol. Let length of park $= x$ m and breadth $= y$ m

$\therefore \quad 2(x + y) = 80 \quad \Rightarrow \quad x + y = 40$ ...(*i*)

and $\quad xy = 400$ ...(*ii*)

From (*i*) and (*ii*), we get

$x(40 - x) = 400 \quad \Rightarrow \quad x^2 - 40x + 400 = 0$

$\therefore \quad D = b^2 - 4ac = (-40)^2 - 4 \times 1 \times 400 = 1600 - 1600 = 0$

The equation has real and equal roots.

$$\therefore \text{ Roots are } \alpha = \frac{-b + \sqrt{D}}{2a} = \frac{-(-40) + \sqrt{0}}{2 \times 1} = \frac{40}{2} = 20$$

$$\text{and} \quad \beta = \frac{-b - \sqrt{D}}{2a} = \frac{-(-40) - \sqrt{0}}{2 \times 1} = \frac{40}{2} = 20.$$

Hence, rectangular park is possible. The length and breadth of the park are equal of 20 m. This rectangle is a square.