7 Coordinate Geometry

Lesson at a Glance

1. The coordinate axes intersect each other at the origin.
2. The coordinates of the origin are (0, 0).
3. The distance of a point from the y-axis is called its x-coordinate (or abscissa).
4. The distance of a point from the x-axis is called its y-coordinate (or ordinate).
5. The coordinates of a point on the x-axis are of the form $(x, 0)$.
6. The coordinates of a point on the y-axis are of the form $(0, y)$.
7. The graph of $ax + by + c = 0$ (a, b are not simultaneously zero) is a straight line.
8. The graph of $y = ax^2 + bx + c$ $(a \neq 0)$ is a parabola.
9. Distance between the points $A(x_1, y_1)$ and $B(x_2, y_2)$ is given by

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

10. The distance of a point $P(x, y)$ from the origin $O(0, 0)$ is

$$OP = \sqrt{x^2 + y^2}.$$

11. The coordinates of the point which divides the line segment joining the points $A(x_1, y_1)$ and $B(x_2, y_2)$, internally, in the ratio $m_1 : m_2$ are

$$\left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right).$$

12. The coordinates of the mid-point of the line segment joining the points $A(x_1, y_1)$ and $B(x_2, y_2)$ are $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

13. Area of a triangle having the vertices A(x_1, y_1), B(x_2, y_2) and C(x_3, y_3) is given by

$$\text{Area of } \Delta ABC = \frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$$

This is the numerical value of the area.

14. If the three points A(x_1, y_1), B(x_2, y_2) and C(x_3, y_3) are collinear, then the area of the triangle formed by these points must be zero,

i.e., $x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) = 0.$

15. The coordinates of the centroid of a triangle formed by the points A(x_1, y_1), B(x_2, y_2) and C(x_3, y_3) are

$$\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right).$$

TEXTBOOK QUESTIONS SOLVED

Exercise 7.1 *(Page – 161-162)*

1. *Find the distance between the following pairs of points:*

(i) *(2, 3), (4, 1)* *(ii)* *(– 5, 7), (– 1, 3)*

(iii) *(a, b), (– a, – b)*

Sol. (*i*) Distance $= \sqrt{(4-2)^2 + (1-3)^2} = \sqrt{4+4}$

$= \sqrt{8} = 2\sqrt{2}$ units.

(*ii*) Distance $= \sqrt{(-1+5)^2 + (3-7)^2} = \sqrt{16+16}$

$= 4\sqrt{2}$ units.

(*iii*) Let points be A(a, b) and B($-a$, $-b$), then

$AB = \sqrt{(-a-a)^2 + (-b-b)^2} = \sqrt{(-2a)^2 + (-2b)^2}$

$= \sqrt{4a^2 + 4b^2} = 2\sqrt{a^2 + b^2}$ units.

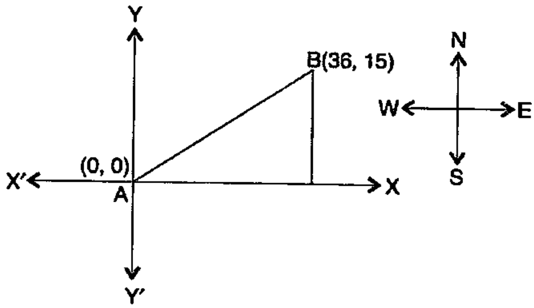
2. *Find the distance between the points (0, 0) and (36, 15). Can you now find the distance between the two towns A and B discussed below as following:*

"A town B is located 36 km east and 15 km north of the town A".

Sol. Distance $= \sqrt{(36-0)^2 + (15-0)^2} = \sqrt{1296 + 225}$

$= \sqrt{1521} = 39.$

Yes, we can find the distance between the two towns as given below.



From figure two towns are situated at A(0, 0) and B(36, 15).

$\therefore$ Distance, AB = 39 km.

3. *Determine if the points (1, 5), (2, 3) and (– 2, – 11) are collinear.*

Sol. Let points be A(1, 5), B(2, 3) and C(–2, –11).

$$AB = \sqrt{(2-1)^2 + (3-5)^2} = \sqrt{1+4} = \sqrt{5};$$

$$BC = \sqrt{(-2-2)^2 + (-11-3)^2} = \sqrt{16+196}$$

$$= \sqrt{212} = 2\sqrt{53};$$

$$AC = \sqrt{(-2-1)^2 + (-11-5)^2} = \sqrt{9+256}$$

$$= \sqrt{265}$$

As we can not get as relation that one distance is equal to sum of other two distances, hence the points are not collinear.

4. *Check whether (5, – 2), (6, 4) and (7, – 2) are the vertices of an isosceles triangle.*

Sol. Let points be A(5, –2), B(6, 4) and C(7, –2).

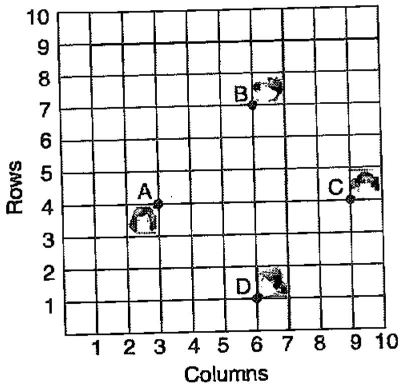
$$AB = \sqrt{(6-5)^2 + (4+2)^2} = \sqrt{1+36} = \sqrt{37}$$

$$BC = \sqrt{(7-6)^2 + (-2-4)^2} = \sqrt{1+36} = \sqrt{37}$$

$$AC = \sqrt{(7-5)^2 + (-2+2)^2} = \sqrt{4+0} = 2$$

As two distances are equal, *i.e.*, AB = BC and one distance is not equal to the sum of others two, hence given points are the vertices of an isosceles triangle.

5. *In a classroom, 4 friends are seated at the points A, B, C and D as shown in figure. Champa and Chameli walk into the class and after observing for a few minutes Champa asks Chameli, "Don't you think ABCD is a square?" Chameli disagrees. Using distance formula, find which of them is correct.*



Sol. According to figure, coordinates of the points are A(3, 4), B(6, 7), C(9, 4) and D(6, 1).

$$\therefore \quad AB = \sqrt{(6-3)^2 + (7-4)^2} = \sqrt{9+9} = 3\sqrt{2}$$

$$BC = \sqrt{(9-6)^2 + (4-7)^2} = \sqrt{9+9} = 3\sqrt{2}$$

$$CD = \sqrt{(6-9)^2 + (1-4)^2} = \sqrt{9+9} = 3\sqrt{2}$$

$$DA = \sqrt{(6-3)^2 + (1-4)^2} = \sqrt{9+9} = 3\sqrt{2}$$

$$AC = \sqrt{(9-3)^2 + (4-4)^2} = \sqrt{36+0} = 6$$

$$BD = \sqrt{(6-6)^2 + (1-7)^2} = \sqrt{0+36} = 6$$

As sides are equal and diagonals are also equal. Hence ABCD is a square. Therefore, Champa is correct.

6. *Name the type of quadrilateral formed, if any, by the following points, and give reasons for your answer:*

(*i*) (– 1, – 2), (1, 0), (– 1, 2), (– 3, 0)

(*ii*) (– 3, 5), (3, 1), (0, 3), (– 1, – 4)

(*iii*) (4, 5), (7, 6), (4, 3), (1, 2)

Sol. (*i*) Let the points be A(– 1, – 2), B(1, 0), C(– 1, 2) and D(– 3, 0) of a quadrilateral ABCD.

$$\therefore \; AB = \sqrt{(1+1)^2+(0+2)^2} = \sqrt{4+4} = 2\sqrt{2}$$

$$BC = \sqrt{(-1-1)^2+(2-0)^2} = \sqrt{4+4} = 2\sqrt{2}$$

$$CD = \sqrt{(-3+1)^2+(0-2)^2} = \sqrt{4+4} = 2\sqrt{2}$$

$$DA = \sqrt{(-1+3)^2+(-2-0)^2} = \sqrt{4+4} = 2\sqrt{2}$$

$$AC = \sqrt{(-1+1)^2+(2+2)^2} = \sqrt{0+16} = 4$$

$$BD = \sqrt{(-3-1)^2+(0-0)^2} = \sqrt{16+0} = 4$$

As sides AB, BC, CD, DA are equal and diagonals AC, BD are also equal. Hence quadrilateral ABCD is a square.

(*ii*) Let the points be A(– 3, 5), B(3, 1), C(0, 3) and D(– 1, – 4).

$$\therefore \; AB = \sqrt{(3+3)^2+(1-5)^2} = \sqrt{36+16}$$

$$= \sqrt{52} = 2\sqrt{13}$$

$$BC = \sqrt{(0-3)^2+(3-1)^2} = \sqrt{9+4} = \sqrt{13}$$

$$CD = \sqrt{(-1-0)^2+(-4-3)^2} = \sqrt{1+49} = \sqrt{50}$$

$$= 5\sqrt{2}$$

$$DA = \sqrt{(-3+1)^2+(5+4)^2} = \sqrt{4+81} = \sqrt{85}$$

$$AC = \sqrt{(0+3)^2+(3-5)^2} = \sqrt{9+4} = \sqrt{13}$$

$$BD = \sqrt{(-1-3)^2+(-4-1)^2} = \sqrt{16+25} = \sqrt{41}$$

Here, AB = AC + BC, *i.e.*, $2\sqrt{13} = \sqrt{13} + \sqrt{13}$, true.

So, A, B and C are collinear.

Hence, no quadrilateral exists.

(*iii*) Let the points be A(4, 5), B(7, 6), C(4, 3) and D(1, 2).

$$\therefore \quad AB = \sqrt{(7-4)^2+(6-5)^2} = \sqrt{9+1} = \sqrt{10}$$

$$BC = \sqrt{(4-7)^2+(3-6)^2} = \sqrt{9+9} = 3\sqrt{2}$$

$$CD = \sqrt{(1-4)^2+(2-3)^2} = \sqrt{9+1} = \sqrt{10}$$

$$DA = \sqrt{(4-1)^2+(5-2)^2} = \sqrt{9+9} = 3\sqrt{2}$$

$$AC = \sqrt{(4-4)^2+(3-5)^2} = \sqrt{0+4} = 2$$

$$BD = \sqrt{(1-7)^2+(2-6)^2} = \sqrt{36+16} = 2\sqrt{13}$$

As AB = CD, BC = DA and AC ≠ BD

Hence, quadrilateral is a parallelogram.

7. *Find the point on the x-axis which is equidistant from (2, – 5) and (– 2, 9).*

Sol. Let P(x, 0) on x-axis be equidistant from A(2, – 5) and B(– 2, 9)

$$\therefore \quad AP = BP \Rightarrow \sqrt{(x-2)^2+(0+5)^2}$$

$$= \sqrt{(x+2)^2+(0-9)^2}$$

$$\Rightarrow \quad x^2 - 4x + 4 + 25 = x^2 + 4x + 4 + 81$$

$$\Rightarrow \quad -8x = 56 \quad \Rightarrow \quad x = -7.$$

$\therefore$ Point is (– 7, 0).

8. *Find the values of y for which the distance between the points P(2, – 3) and Q(10, y) is 10 units.*

Sol. $\sqrt{(10-2)^2+(y+3)^2} = 10 \Rightarrow 64 + 9 + 6y + y^2 = 100$

$$\Rightarrow \quad y^2 + 6y - 27 = 0 \Rightarrow (y+9)(y-3) = 0$$

$$\Rightarrow \quad y = -9, 3.$$

9. *If Q(0, 1) is equidistant from P(5, – 3) and R(x, 6), find the values of x. Also find the distances QR and PR.*

Sol. As Q(0, 1) is equidistant from P(5, – 3) and R(x, 6).

$$\therefore \quad PQ = RQ$$

$\Rightarrow \quad \sqrt{(0-5)^2+(1+3)^2} = \sqrt{(0-x)^2+(1-6)^2}$

Squaring and simplifying,

$$25 + 16 = x^2 + 25$$

$\Rightarrow \quad x^2 = 16 \quad \Rightarrow \quad x = \pm 4$

$\therefore$ Point R is R(4, 6) or R(– 4, 6).

With point R(4, 6),

$$QR = \sqrt{(4-0)^2+(6-1)^2} = \sqrt{16+25} = \sqrt{41}$$

$$PR = \sqrt{(4-5)^2+(6+3)^2} = \sqrt{1+81} = \sqrt{82}$$

With point R(– 4, 6),

$$QR = \sqrt{(-4-0)^2+(6-1)^2} = \sqrt{16+25} = \sqrt{41}$$

$$PR = \sqrt{(-4-5)^2+(6+3)^2} = \sqrt{81+81} = 9\sqrt{2}.$$

10. *Find a relation between x and y such that the point (x, y) is equidistant from the points (3, 6) and (– 3, 4).*

Sol. If P(x, y) is equidistant from the points A(3, 6) and B(– 3, 4), then

$$AP = BP$$

$\Rightarrow \quad \sqrt{(x-3)^2+(y-6)^2} = \sqrt{(x+3)^2+(y-4)^2}$

$\Rightarrow \quad x^2 - 6x + 9 + y^2 - 12y + 36 = x^2 + 6x + 9 + y^2 - 8y + 16$

$\Rightarrow \quad -12x - 4y + 20 = 0$

$\Rightarrow \quad 3x + y - 5 = 0$ is the required relation.

Exercise 7.2 *(Page – 167)*

1. *Find the coordinates of the point which divides the join of (– 1, 7) and (4, – 3) in the ratio 2 : 3.*

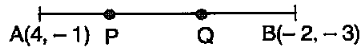
Sol. Coordinates of point are $\left(\frac{8-3}{2+3}, \frac{-6+21}{2+3}\right)$, *i.e.*, (1, 3).

2. *Find the coordinates of the points of trisection of the line segment joining (4, – 1) and (– 2, – 3).*

Sol. Let P and Q be the points of trisection.

P divides AB in the ratio 1 : 2.

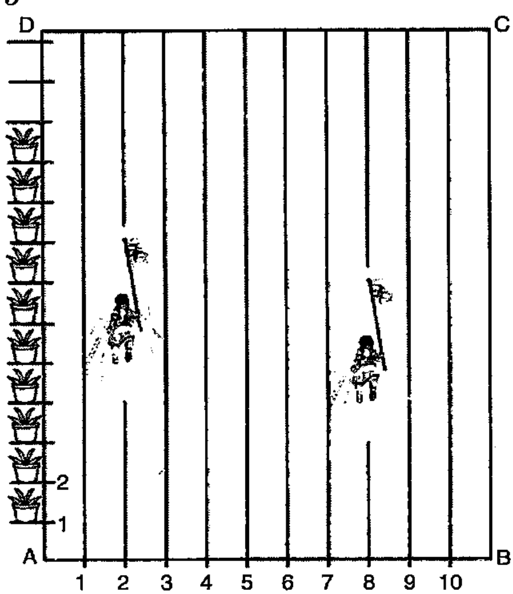
Hence, coordinates of P are



$P\left(\dfrac{-2+8}{1+2}, \dfrac{-3-2}{1+2}\right)$, *i.e.*, $P\left(2, \dfrac{-5}{3}\right)$ and Q divides AB in the ratio 2 : 1.

Hence, coordinates of Q are $Q\left(\dfrac{-4+4}{3}, \dfrac{-6-1}{3}\right)$, *i.e.*, $Q\left(0, \dfrac{-7}{3}\right)$.

3. *To conduct Sports Day activities, in your rectangular shaped school ground ABCD, lines have been drawn with chalk powder at a distance of 1m each. 100 flower pots have been placed at a distance of 1m from each other along AD, as shown in figure. Niharika runs $\frac{1}{4}$th the distance AD on the 2nd line and posts a green flag. Preet runs $\frac{1}{5}$th the distance AD on the eighth line and posts a*



red flag. What is the distance between both the flags? If

Rashmi has to post a blue flag exactly halfway between the line segment joining the two flags, where should she post her flag?

Sol. Position of green flag is G(2, 25).

Position of red flag is R(8, 20).

$\therefore$ Distance between these two flags,

i.e., $GR = \sqrt{(8-2)^2 + (20-25)^2}$ m $= \sqrt{36+25} = \sqrt{61}$ m.

Let Rashmi has to post a blue flag at B mid-way of GR.

Mid-point of GR is $\left(\frac{2+8}{2}, \frac{25+20}{2}\right)$, *i.e.,* (5, 22.5)

Hence, Rashmi has to move in 5th line at a distance of 22.5 m.

4. *Find the ratio in which the line segment joining the points (– 3, 10) and (6, – 8) is divided by (– 1, 6).*

Sol. Let the ratio be $k : 1$,

then $\left(\frac{6k-3}{k+1}, \frac{-8k+10}{k+1}\right) = (-1, 6)$.

$\Rightarrow \quad \frac{6k-3}{k+1} = -1 \quad$ and $\quad \frac{-8k+10}{k+1} = 6$

$\Rightarrow \quad 6k - 3 = -k - 1 \quad$ and $\quad -8k + 10 = 6k + 6$

$\Rightarrow \quad 7k = 2 \quad$ and $\quad -14k = -4$

$\Rightarrow \quad k = \frac{2}{7}$ in both cases.

Therefore, the required ratio is $\frac{2}{7} : 1$, *i.e.,* 2 : 7.

5. *Find the ratio in which the line segment joining A(1, – 5) and B(– 4, 5) is divided by the x-axis. Also find the coordinates of the point of division.*

Sol. Let ratio be $k : 1$, then point of division is $\left(\frac{-4k+1}{k+1}, \frac{5k-5}{k+1}\right)$.

If this point lies on x-axis, then $y = 0 \Rightarrow \frac{5k-5}{k+1} = 0$

$\Rightarrow \quad k = 1$

$\therefore$ Ratio is 1 : 1 and point of division is $\left(\frac{-4+1}{2}, 0\right)$, *i.e.*, $\left(\frac{-3}{2}, 0\right)$.

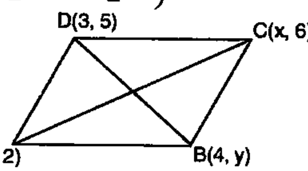
6. *If (1, 2), (4, y), (x, 6) and (3, 5) are the vertices of a parallelogram taken in order, find x and y.*

Sol. We know that diagonals of a parallelogram bisect each other.

Therefore, $\left(\frac{1+x}{2}, \frac{2+6}{2}\right) = \left(\frac{3+4}{2}, \frac{5+y}{2}\right)$

$\Rightarrow \quad \frac{1+x}{2} = \frac{7}{2}$ and $4 = \frac{5+y}{2}$

$\Rightarrow \quad x = 6$ and $y = 3$.

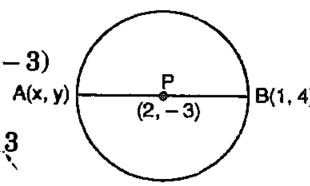


7. *Find the coordinates of a point A, where AB is the diameter of a circle whose centre is (2, – 3) and B is (1, 4).*

Sol. Centre P is mid-point of AB.

Therefore, $\left(\frac{x+1}{2}, \frac{y+4}{2}\right) = (2, -3)$

$\Rightarrow \quad \frac{x+1}{2} = 2$ and $\frac{y+4}{2} = -3$



$\Rightarrow \quad x = 3$ and $y = -10$. Hence coordinates of A are $(3, -10)$.

8. *If A and B are (– 2, – 2) and (2, – 4) respectively, find the coordi-nates of P such that AP = $\frac{3}{7}$AB and P lies on the line segment AB.*

Sol. $AP = \frac{3}{7} AB \quad \Rightarrow \quad 7AP = 3(AP + PB)$

$\Rightarrow \quad 4AP = 3PB \quad \Rightarrow \quad \frac{AP}{PB} = \frac{3}{4}$

i.e., $AP : PB = 3 : 4$

$\therefore$ Coordinates of P are $\left(\frac{6-8}{3+4}, \frac{-12-8}{3+4}\right)$,

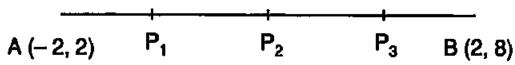
i.e., $\left(\frac{-2}{7}, \frac{-20}{7}\right)$.

9. *Find the coordinates of the points which divide the line segment joining A(– 2, 2) and B(2, 8) into four equal parts.*

Sol. Here, the given points are:

A (– 2, 2) and B (2, 8)

Let P_1, P_2 and P_3 divide AB in four equal parts.



$\because \quad AP_1 = P_1P_2 = P_2P_3 = P_3B$

Obviously, P_2 is the mid point of AB

$\therefore$ Coordinates of P_2 are:

$$\left(\frac{-2+2}{2}, \frac{2+8}{2}\right) \text{ or } (0, 5)$$

Again, P_1 is the mid point of AP_2.

$\therefore$ Coordinates of P_1 are:

$$\left(\frac{-2+0}{2}, \frac{2+5}{2}\right) \text{ or } \left(-1, \frac{7}{2}\right)$$

Also P_3 is the mid point of P_2B.

$\therefore$ Coordinates of P_3 are:

$$\left(\frac{0+2}{2}, \frac{5+8}{2}\right) \text{ or } \left(1, \frac{13}{2}\right)$$

Thus, the coordinates of P_1, P_2 and P_3 are:

(0, 5), $\left(-1, \frac{7}{2}\right)$ and $\left(1, \frac{13}{2}\right)$ respectively.

10. *Find the area of a rhombus if its vertices are (3, 0), (4, 5), (– 1, 4) and (– 2, – 1) taken in order.*

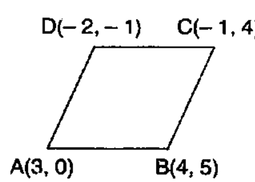
[**Hint:** *Area of a rhombus* = $\frac{1}{2}$ *(product of its diagonals)*]

Sol. $AC = \sqrt{(-1-3)^2 + (4-0)^2} = \sqrt{16+16} = 4\sqrt{2}$

$BD = \sqrt{(-2-4)^2 + (-1-5)^2} = \sqrt{36+36} = 6\sqrt{2}$

$\therefore$ Area of rhombus $= \frac{1}{2} \times AC \times BD$

$= \frac{1}{2} \times 4\sqrt{2} \times 6\sqrt{2} = 24$ sq. units.



Exercise 7.3 *(Page – 170)*

1. *Find the area of the triangle whose vertices are:*

(*i*) (2, 3), (– 1, 0), (2, – 4)   (*ii*) (– 5, – 1), (3, – 5), (5, 2)

Sol. (*i*) Area of $\Delta = \frac{1}{2}[2(0 + 4) - 1(-4 - 3) + 2(3 - 0)]$

$= \frac{1}{2}[8 + 7 + 6] = \frac{21}{2}$ sq. units.

(*ii*) Area of $\Delta = \frac{1}{2}[-5(-5 - 2) + 3(2 + 1) + 5(-1 + 5)]$

$= \frac{1}{2}[35 + 9 + 20] = 32$ sq. units.

2. *In each of the following find the value of 'k', for which the points are collinear.*

(*i*) (7, – 2), (5, 1), (3, *k*)   (*ii*) (8, 1), (*k*, – 4), (2, – 5)

Sol. (*i*) If points are collinear, then area of triangle = 0

$\Rightarrow \frac{1}{2}[7(1 - k) + 5(k + 2) + 3(-2 - 1)] = 0$

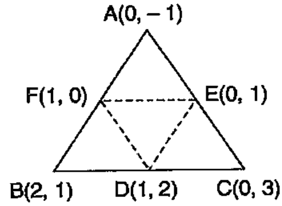
$\Rightarrow \frac{1}{2}[7 - 7k + 5k + 10 - 9] = 0$

$\Rightarrow -2k + 8 = 0 \quad \Rightarrow \quad k = 4.$

(*ii*) $\Rightarrow \frac{1}{2}[8(-4 + 5) + k(-5 - 1) + 2(1 + 4)] = 0$

$\Rightarrow 8 - 6k + 10 = 0 \Rightarrow 6k = 18 \Rightarrow k = 3.$

3. *Find the area of the triangle formed by joining the mid-points of the sides of the triangle whose vertices are (0, – 1), (2, 1) and (0, 3). Find the ratio of this area to the area of the given triangle.*



Sol. $ar(\Delta ABC) = \frac{1}{2}[0(1 - 3) + 2(3 + 1)$

$+ 0(-1 - 1)]$ sq. units.

$= 4$ sq. units...(*i*)

Also coordinates of mid-points of sides are D(1, 2), E(0, 1) and F(1, 0)

$\therefore \quad ar(\Delta DEF) = \frac{1}{2}[1(1-0) + 0(0-2) + 1(2-1)]$ sq. units

$= 1$ sq. unit ...(*ii*)

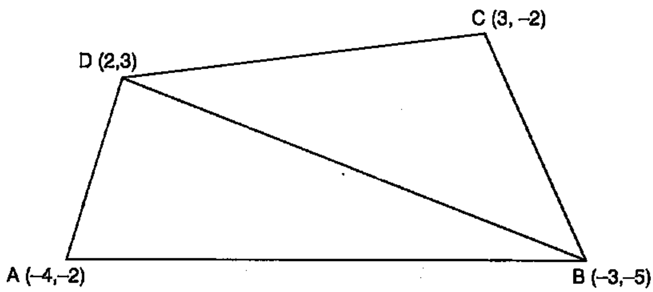
From (*i*) and (*ii*), we have

$ar(\Delta ABC) : ar(\Delta DEF) = 4 : 1.$

4. *Find the area of the quadrilateral whose vertices, taken in order, are (– 4, – 2), (– 3, – 5), (3, – 2) and (2, 3).*

Sol. Let A (– 4, – 2), B (– 3, – 5), C (3, – 2) and D (2, 3) be the vertices of the quadrilateral.

Let us join diagonal BD.



Now, **ar (Δ ABD)** $= \frac{1}{2}[(-4)\{-5-3\} + (-3)\{3-(-2)\} + 2\{(-2)-(-5)\}]$

$= \frac{1}{2}[(-4)(-8) + (-3)(5) + 2(-2+5)]$

$= \frac{1}{2}[32 + (-15) + 6]$

$= \frac{1}{2}[-33] = \frac{23}{2}$ sq. units

ar (Δ CBD) $= \frac{1}{2}[3(-5-3) + (-3)\{3-(-2)\} + 2\{(-2)-(-5)\}]$

$= \frac{1}{2}[3(-8) + -3(5) + 2(3)]$

$= \frac{1}{2}[-24 - 15 + 6]$

$$= \frac{1}{2}[-33] = \frac{33}{2} \text{ sq. units, (numerically)}$$

Since, ar (quad ABCD) = ar (Δ ABD) + ar Δ CBD

∴ ar (quad ABCD) $= \left(\frac{23}{2} + \frac{33}{2}\right)$ sq. units

$$= \frac{56}{2} \text{ sq. units} = 28 \text{ sq. units.}$$

5. *You have studied in Class IX, (Chapter 9), that a median of a triangle divides it into two triangles of equal areas. Verify this result for ΔABC whose vertices are A(4, – 6), B(3, – 2) and C(5, 2).*

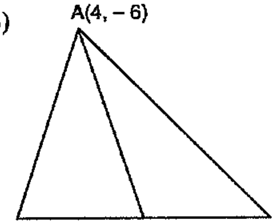
Sol. Let AD be median of ΔABC. Then D is mid-point of BC.

Therefore, coordinates of D are $\left(\frac{5+3}{2}, \frac{-2+2}{2}\right)$, *i.e.*, (4, 0).

$$ar(\Delta ABD) = \left|\frac{1}{2}[4(-2-0) + 3(0+6) + 4(-6+2)]\right|$$

$$= \left|\frac{1}{2}[-8 + 18 - 16]\right| = |-3| = 3 \text{ sq. units.}$$

$$ar(\Delta ADC) = \left|\frac{1}{2}[4(0-2) + 4(2+6) + 5(-6-0)]\right|$$



$$= \frac{1}{2}|-8 + 32 - 30|$$

$$= \frac{1}{2}|-6| = 3 \text{ sq. units.}$$

Hence, $ar(\Delta ABD) = ar(\Delta ADC)$. **Hence proved.**

Exercise 7.4 (OPTIONAL) *(Page – 171-172)*

1. *Determine the ratio in which the line 2x + y – 4 = 0 divides the line segment joining the points A(2, – 2) and B(3, 7).*

Sol. Let the ratio be $k : 1$. Then coordinates of point of division are $\left(\frac{3k+2}{k+1}, \frac{7k-2}{k+1}\right)$.

This point must lie on the line $2x + y - 4 = 0$, so we have

$$2\left(\frac{3k+2}{k+1}\right) + \left(\frac{7k-2}{k+1}\right) - 4 = 0$$

$$\Rightarrow \quad 6k + 4 + 7k - 2 - 4k - 4 = 0 \Rightarrow 9k - 2 = 0$$

$$\Rightarrow \quad k = \frac{2}{9}$$

Therefore, the required ratio is $\frac{2}{9} : 1$, *i.e.*, $2 : 9$.

2. *Find a relation between x and y if the points (x, y), (1, 2) and (7, 0) are collinear.*

Sol. If (x, y), $(1, 2)$ and $(7, 0)$ are collinear, then the area of triangle formed by these points must be zero.

Area of $\Delta = 0$

$$\Rightarrow \quad \frac{1}{2}[x(2-0) + 1(0-y) + 7(y-2)] = 0$$

$$\Rightarrow \quad 2x - y + 7y - 14 = 0 \Rightarrow 2x + 6y - 14 = 0$$

$\Rightarrow \quad x + 3y - 7 = 0$ is the required relation.

3. *Find the centre of a circle passing through the points (6, – 6), (3, – 7) and (3, 3).*

Sol. Let $P(x, y)$ be the centre of the circle passing through $A(6, -6)$, $B(3, -7)$ and $C(3, 3)$. Then by definition,

$$AP = BP = CP$$

$$\Rightarrow \sqrt{(x-6)^2 + (y+6)^2} = \sqrt{(x-3)^2 + (y+7)^2}$$

$$= \sqrt{(x-3)^2 + (y-3)^2} \qquad ...(i)$$

From (i), consider, $\sqrt{(x-3)^2 + (y+7)^2}$

$$= \sqrt{(x-3)^2 + (y-3)^2}$$

Squaring and simplifying, we get

$$x^2 - 6x + 9 + y^2 + 14y + 49 = x^2 - 6x + 9 + y^2 - 6y + 9$$

$$\Rightarrow \quad 20y = -40 \Rightarrow y = -2 \qquad ...(ii)$$

Again from (i), consider,

$$\sqrt{(x-6)^2 + (y+6)^2} = \sqrt{(x-3)^2 + (y+7)^2}$$

Squaring and simplifying, we get

$$x^2 - 12x + 36 + y^2 + 12y + 36 = x^2 - 6x + 9 + y^2 + 14y + 49$$

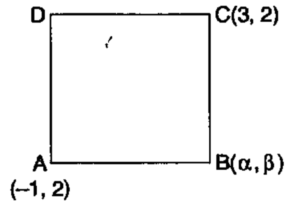
$\Rightarrow\ -6x - 2y + 14 = 0 \quad \Rightarrow\ 3x + y - 7 = 0$

Substituting the value for y from (*ii*), we get

$3x - 2 - 7 = 0 \quad \Rightarrow\ 3x = 9 \quad \Rightarrow\ x = 3$

Hence, the centre is P(3, – 2).

4. *The two opposite vertices of a square are (– 1, 2) and (3, 2). Find the coordinates of the other two vertices.*



Sol. Let the coordinates of point B of the square ABCD be (α, β)

$$AC = \sqrt{(3+1)^2 + (2-2)^2} = \sqrt{16} = 4$$

$$\therefore \qquad AB = BC = \frac{4}{\sqrt{2}} = 2\sqrt{2}$$

Also, $AB = \sqrt{(\alpha+1)^2 + (\beta-2)^2} = 2\sqrt{2}$

$\Rightarrow\ \alpha^2 + 2\alpha + 1 + \beta^2 - 4\beta + 4 = 8$

$\Rightarrow\ \alpha^2 + \beta^2 + 2\alpha - 4\beta = 3$ ...(*i*)

And $BC = \sqrt{(\alpha-3)^2 + (\beta-2)^2} = 2\sqrt{2}$

On simplifying, we have

$\alpha^2 + \beta^2 - 6\alpha - 4\beta = -5$ ...(*ii*)

Subtracting (*ii*) from (*i*) and simplifying, we obtain

$\alpha = 1$

Substituting $\alpha = 1$ in (*i*) and simplifying, we obtain

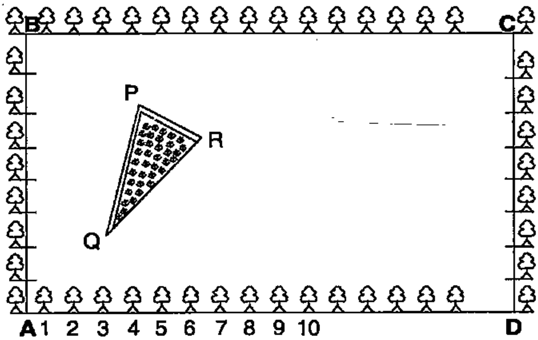
$\beta = 0, 4$

Hence, the coordinates of other two vertices are (1, 0) and (1, 4).

5. *The Class X students of a secondary school in Krishinagar have been allotted a rectangular plot of land for their gardening activity. Saplings of Gulmohar are planted on the boundary at a distance of 1m from each other. There is a triangular grassy lawn in the plot as shown in the figure. The students are to sow seeds of flowering plants on the remaining area of the plot.*

   (*i*) *Taking A as origin, find the coordinates of the vertices of the triangle.*

   (*ii*) *What will be the coordinates of the vertices of ΔPQR if C is the origin?*



Also calculate the areas of the triangles in these cases. What do you observe?

Sol. (*i*) P(4, 6), Q(3, 2), R(6, 5)

(*ii*) P(– 12, – 2), Q(– 13, – 6), R(– 10, – 3)

Area of ΔPQR in case (*i*)

$$= \left|\frac{1}{2}[4(2-5)+3(5-6)+6(6-2)]\right| \text{ sq. units}$$

$$= \left|\frac{1}{2}[-12-3+24]\right| \text{ sq. units} = \frac{9}{2} \text{ sq. units.}$$

6. *The vertices of a ΔABC are A(4, 6), B(1, 5) and C(7, 2). A line is drawn to intersect sides AB and AC at D and E respectively, such that* $\frac{AD}{AB} = \frac{AE}{AC} = \frac{1}{4}$. *Calculate the area of the ΔADE and compare it with the area of ΔABC. (Recall "The converse of Basic Proportionality Theorem" and 'Theorem of Similar Triangles taking their Areas and Corresponding Sides').*

Sol. We have $\frac{AD}{AB} = \frac{1}{4}$

$$\Rightarrow \quad \frac{AB}{AD} = \frac{4}{1}$$

$$\Rightarrow \quad \frac{AD + DE}{AD} = \frac{4}{1}$$

$$\Rightarrow \quad \frac{AD}{AD} + \frac{DE}{AD} = \frac{4}{1} = 1 + \frac{3}{1}$$

$\Rightarrow \quad 1+\frac{DE}{AD}=1+\frac{3}{1} \quad \Rightarrow \quad \frac{DE}{AD}=\frac{3}{1}$

$\Rightarrow \quad AD : DE = 1 : 3$

Thus, the point D divides AB in the ratio 1 : 3

$\therefore$ **The coordinates of D are:**

$$\left[\frac{(1\times1)+(3\times4)}{1+3}, \frac{(1\times5)+(3\times6)}{1+3}\right]$$

or $\left[\frac{1+12}{4}, \frac{5+18}{4}\right]$

or $\left(\frac{13}{4}, \frac{23}{4}\right)$

Similarly, AE : EC = 1 : 3

i.e., E divides AC in the ratio 1 : 3

$\Rightarrow$ **Coordinates of E are:**

$$\left[\frac{(1\times7)+(3\times4)}{1+3}, \frac{1\times2+3\times6}{1+3}\right]$$

or $\left[\frac{7+12}{4}, \frac{2+18}{4}\right]$

or $\left[\frac{19}{4}, 5\right]$

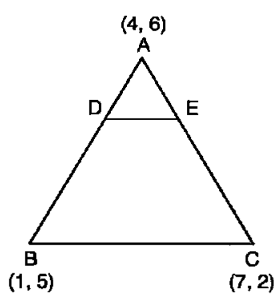
Now, **ar (Δ ADE)**

$$= \frac{1}{2}\left[4\left(\frac{23}{4}-5\right)+\frac{13}{4}(5-6)+\frac{19}{4}\left(6-\frac{23}{4}\right)\right]$$

$$= \frac{1}{2}\left[(23-20)+\frac{13}{4}(1)+\frac{19}{4}\left(\frac{24-23}{4}\right)\right]$$

$$= \frac{1}{2}\left(3-\frac{13}{4}+\frac{19}{16}\right)$$

$$= \frac{1}{2}\left[\frac{48+52+19}{16}\right] = \frac{15}{32} \text{ sq. units.}$$



Area of Δ ABC

$$= \frac{1}{2}\ [4\ (5-2)+1\ (2-6)+7\ (6-5)]$$

$$= \frac{1}{2} [(4 \times 3) + 1 \times (-4) + 7 \times 1]$$

$$= \frac{1}{2} [12 + (-4) + 7]$$

$$= \frac{1}{2}(15) = \frac{15}{2} \text{ sq. units.}$$

Now, $$\frac{\text{ar}(\Delta \text{ADE})}{\text{ar}(\Delta \text{ABC})} = \frac{\frac{15}{32}}{\frac{15}{2}} = \frac{15}{32} \times \frac{2}{15} = \frac{1}{16}$$

$\Rightarrow$ ar (Δ ADE) : ar (Δ ABC) = **1 : 16.**

[Use the result 'if two triangles are similar, then their areas are proportional to the squares of their corresponding sides.']

7. *Let A(4, 2), B(6, 5) and C(1, 4) be the vertices of Δ ABC.*

(*i*) *The median from A meets BC at D. Find the coordinates of the point D.*

(*ii*) *Find the coordinates of the point P on AD such that AP : PD = 2 : 1.*

(*iii*) *Find the coordinates of points Q and R on medians BE and CF respectively such that BQ : QE = 2 : 1 and CR : RF = 2 : 1.*

(*iv*) *What do you observe?*

[**Note:** *The point which is common to all the three medians is called the centroid and this point divides each median in the ratio 2 : 1.*]

(*v*) *If A(x_1, y_1), B(x_2, y_2) and C(x_3, y_3) are the vertices of Δ ABC, find the coordinates of the centroid of the triangle.*

Sol. (*i*) D is mid-point of BC, so coordinates of D are

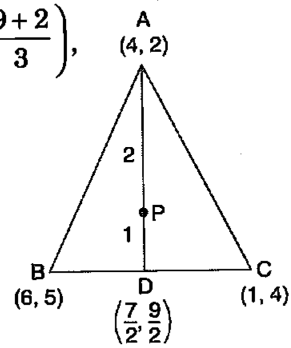
$$D\left(\frac{6+1}{2}, \frac{5+4}{2}\right) \quad i.e., \quad D\left(\frac{7}{2}, \frac{9}{2}\right)$$

(*ii*) AP : PD = 2 : 1

P divides AD in the ratio 2 : 1.

Coordinates of P are $P\left(\frac{7+4}{3}, \frac{9+2}{3}\right)$,

i.e., $P\left(\frac{11}{3}, \frac{11}{3}\right)$.



(*iii*) E is mid-point of AC. Coordinates of E are

$E\left(\frac{5}{2}, 3\right)$.

Q divides BE in the ratio 2 : 1.

Coordinates of Q are $Q\left(\frac{5+6}{3}, \frac{6+5}{3}\right)$, *i.e.*, $Q\left(\frac{11}{3}, \frac{11}{3}\right)$

Similarly, we notice coordinates of R are $R\left(\frac{11}{3}, \frac{11}{3}\right)$.

(*iv*) P, Q and R are the same point.

All the three medians of a triangle meet at a unique point, which is called the centroid and this point divides each of the three medians in the ratio 2 : 1.

(*v*) Let AD be a median drawn from the vertex A of the ΔABC to the base BC, which meets BC at D.

Coordinates of D are $\left(\frac{x_2+x_3}{2}, \frac{y_2+y_3}{2}\right)$

The centroid of ΔABC divides AD in the ratio 2 : 1. Therefore, the coordinates of centroid are

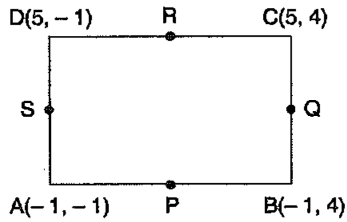
$$\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right).$$

8. *ABCD is a rectangle formed by the points A(–1, –1), B(–1, 4), C(5, 4) and D(5, –1). P, Q, R and S are the mid-points of AB, BC, CD and DA respectively. Is the quadrilateral PQRS a square? a rectangle? or a rhombus? Justify your answer.*

Sol. Coordinate of P, Q, R and S are $\left(-1, \frac{3}{2}\right)$, (2, 4), $\left(5, \frac{3}{2}\right)$ and (2, –1) respectively.

$$PQ = \sqrt{(2+1)^2 + \left(4 - \frac{3}{2}\right)^2}$$

$$= \sqrt{9 + \frac{25}{4}} = \frac{\sqrt{61}}{2}$$



$$QR = \sqrt{(5-2)^2 + \left(\frac{3}{2} - 4\right)^2}$$

$$= \sqrt{9 + \frac{25}{4}} = \frac{\sqrt{61}}{2}$$

$$RS = \sqrt{(2-5)^2 + \left(-1 - \frac{3}{2}\right)^2} = \sqrt{9 + \frac{25}{4}} = \frac{\sqrt{61}}{2}$$

$$SP = \sqrt{(2+1)^2 + \left(-1 - \frac{3}{2}\right)^2} = \sqrt{9 + \frac{25}{4}} = \frac{\sqrt{61}}{2}$$

$$PR = \sqrt{(5+1)^2 + \left(\frac{3}{2} - \frac{3}{2}\right)^2} = \sqrt{36+0} = 6$$

$$QS = \sqrt{(2-2)^2 + (4+1)^2} = \sqrt{0+25} = 5$$

As PQ = QR = RS = SP and PR ≠ QS.

Hence, PQRS is a rhombus.